Syllabus: Allometric Scaling, Complex Networks, and Universal Dynamics

General Course (Interdisciplinary Advanced Undergraduate & Graduate)

University of Jan Kochanowski (UJK), Kielce

November 5, 2026

Course Overview

Course Duration: 10 Hours Total (5 Modules × 2 Hours)
Target Audience: Advanced Undergraduate and Graduate Students in Physics, Chemistry, Biology, Mathematics, Economics, and Social Sciences.
Main Reference Text: *Thermofractals, Dynamics in Fractal Spaces, and Applications* (Part II: Applications).

Course Abstract

This course provides a comprehensive conceptual introduction to the science of scaling, self-similarity, and complex network dynamics across biological, epidemiological, and urban systems. Students explore how simple space-filling geometric constraints, physical transport laws, and network topologies give rise to universal power-law dynamics—from organismal metabolic rates to epidemic growth and the non-linear growth of megacities.

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Module Breakdown & Content

Module 1: Foundations of Scale, Geometry, and Scaling Taxonomies (Hours 1–2)

Beamer Alignment: Slides 1–5 (Recap & Concept of Scale)
Course Notes Alignment: Section 1.1 & Section 8.1

- **Concept of Scale and Self-Similarity:** Topological versus fractional dimensions; self-similar structures across biological and physical systems.
- **Isometric vs. Allometric Scaling:** Proportional geometric scaling (isometry) versus differential structural growth under scale variations (allometry).
- **Logarithmic Data Analysis:** Log-log representations as diagnostic tools for identifying power laws and extracting scaling exponents from empirical measurements.
- **Biomechanical Applications:** Size invariance of jump height in animals resulting from muscle force versus displacement scaling; biomechanical safety constraints and positive leg-bone allometry in massive quadrupeds and dinosaurs.

Module 2: Biological Allometry, Body Proportions, and the WBE Network Model (Hours 3–4)

Beamer Alignment: Slides 6–12 (Biological Scaling Laws)
Course Notes Alignment: Chapter 8 & Chapter 9

- **Kleiber's Law vs. Euclidean Predictions:** Contrast between classical Euclidean heat-loss models and empirical quarter-power metabolic scaling across animal species.
- **Cylindrical Body Model and Body Mass Index:** Modeling body geometry to derive volume-to-surface area ratios; scale invariance of body proportions and physical interpretation of the Body Mass Index as a thermal/structural load index.
- **West-Brown-Enquist (WBE) Model Postulates:** Hierarchical distribution networks; space-filling geometry, invariance of terminal exchange units, and energy minimization principles.
- **Hydrodynamics and Flow Dynamics:** Viscous flow optimization (Murray's Law) in small vessels versus pulsatile wave propagation (Womersley dynamics) in major elastic arteries; the steplike crossover in vessel branching ratios.
- **Biological Pace of Life:** Invariant lifetime heartbeats across mammalian species and mass-specific metabolic acceleration in small organisms.

Module 3: Hierarchical Networks, Social Contact, and Epidemic Kinetics (Hours 5–6)

Beamer Alignment: Slides 13–15 (Epidemic Super-Linearity & Fractal Contacts)
Course Notes Alignment: Chapter 11

- **Hierarchical Network Topology:** Emergence of scale-free behavior from local connectivity constraints; symmetry breaking at the individual interaction level.
- **Non-Extensive Information and Pathogen Propagation:** Combinatorial transmission pathways in small groups; transition from local contact rules to continuous non-extensive propagation kinetics.
- **Generalized Fractal SIR (fSIR) Model:** Rate equations with non-linear contact dependencies; sub-exponential and power-law early growth dynamics in structured populations.
- **Recovery of Classical SIR Dynamics:** Classical homogeneous mixing models framed as asymptotic limits of unconstrained spatial interaction.
- **Empirical Validation and Mobility Mapping:** Decoupling viral characteristics from population behavior; mobile tracking metrics, contact coordination numbers, and geographical micro-clustering of infection chains.

Module 4: Urban Science, Socioeconomic Systems, and Economic Production (Hours 7–8)

Beamer Alignment: Slides 16–19 (Urban Scaling Laws & Innovation)
Course Notes Alignment: Chapter 12 (Sections 12.1–12.4)

- **Taxonomy of Urban Exponents:** Sub-linear infrastructure scaling (economies of scale), linear individual maintenance scaling, and super-linear socioeconomic scaling (increasing returns to scale).

- **Models of Urban Mixing:** Santa Fe Institute (SFI) spatial social mixing networks versus scale-free non-extensive kinetic views of urban agglomeration.
- **Micro-Foundations of Urban Output:** Aggregate output identities; emergence of Cobb-Douglas production functions and scale-dependent Total Factor Productivity.
- **Scale-Adjusted Metropolitan Indicators (SAMIs):** Isolating size-independent local city performance using logarithmic residuals; non-Gaussian (Laplace) residual statistics and multi-decadal performance persistence.

Module 5: Growth Dynamics, Singularities, and Course Unification (Hours 9–10)

Beamer Alignment: Slides 20–21 (Foundations & Key Takeaways)
Course Notes Alignment: Chapter 12 (Sections 12.5–12.6)

- **Non-Linear Urban Growth Equations:** Resource allocation between maintenance costs and new growth under super-linear feedback.
- **Finite-Time Singularities:** Unbounded super-linear growth dynamics leading to infinite resource demand in finite time.
- **Accelerating Innovation Cycles:** Continuous technological reset requirements; contracting cycle durations necessary to prevent urban collapse.
- **Grand Unification of Scaling Principles:** Comparative analysis of hierarchical space-filling structures in biological transport, urban socioeconomic networks, and epidemic contact kinetics.

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Key Bibliography & References

1. Mandelbrot, B. B. (1983). *The Fractal Geometry of Nature*. W. H. Freeman and Company.

2. Kleiber, M. (1932). Body size and metabolism. *Hilgardia*, 6(11), 315–353. https://doi.org/10.3733/hilg.v06n11p315

3. West, G. B., Brown, J. H., & Enquist, B. J. (1997). A general model for the origin of allometric scaling laws in biology. *Science*, 276(5309), 122–126. https://www.science.org/doi/10.1126/science.276.5309.122

4. West, G. B., Savage, V. M., Gillooly, J., Enquist, B. J., Woodruff, W. H., & Brown, J. H. (2003). Why does metabolic rate scale with body size? *Nature*, 421(6924), 713–713. https://www.nature.com/articles/421713a

5. Bettencourt, L. M., Lobo, J., Helbing, D., Kühnert, C., & West, G. B. (2007). Growth, innovation, scaling, and the pace of life in cities. *Proceedings of the National Academy of Sciences*, 104(17), 7301–7306. https://www.pnas.org/doi/10.1073/pnas.0610172104

6. Bettencourt, L. M., Lobo, J., Strumsky, D., & West, G. B. (2010). Urban scaling and its deviations: Revealing the structure of cities, wealth, and innovation. *PLoS ONE*, 5(11), e13541. https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0013541

7. Lobo, J., Bettencourt, L. M., Strumsky, D., & West, G. B. (2013). Urban scaling and the production function for cities. *PLoS ONE*, 8(3), e58407.
https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0058407

8. Tsallis, C. (1988). Possible generalization of Boltzmann-Gibbs statistics. *Journal of Statistical Physics*, 52(1), 479–487.
https://link.springer.com/article/10.1007/BF01016429

9. Deppman, A., & Andrade-II, E. O. (2021). Emergence of Tsallis statistics in fractal networks. *PLoS ONE*, 16(9), e0257855.
https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0257855

10. Abbasi, M., Andrade-II, E. O., Bollini, A. L., Castillo, J. L. B., Deppman, A., et al. (2020). Fractal signatures of the COVID-19 spread. *Chaos, Solitons & Fractals*, 140, 110119.
https://doi.org/10.1016/j.chaos.2020.110119